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# AUTOMATED DESIGN OF THE EUROPA ORBITER TOUR

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In this paper we investigate tours involving the Jovian satellites Europa, Ganymede, and Callisto for the Europa Orbiter mission. The principal goal of the tour design is to lower arrival  $V_{\infty}$  at the final Europa encounter while meeting all of the design constraints. Key constraints arise from considering the total time of flight and the radiation dosage of a tour. These tours may employ 14 or more encounters with the Jovian satellites, hence there is an enormous number of possible sequences of these satellites to investigate. We develop a graphical method that greatly aids the design process and present the best tours found so far.

## Introduction

THE Europa Orbiter mission is currently scheduled to arrive at Jupiter by the end of the decade. (At the time of this writing, the launch date has slipped to 2006. Although our results are based on a 2003 launch date, our techniques are generally applicable to all future tour designs). The mission will investigate the possibility that liquid oceans may exist beneath the surface ice of Europa. It will attempt to map these regions of liquid water for follow-up missions to Europa. The recent discovery of life in the ice of Lake Vostok, a lake deep beneath the Antarctic ice cap, lends impetus to Europa missions with the suggestion that life may be possible on Europa.

In order to orbit Europa, the arrival  $V_{\infty}$  must be reduced as much as possible prior to orbit insertion. In this paper we investigate the problem of lowering the arrival  $V_{\infty}$  with a tour (i.e. a sequence of gravity assists) of the Jovian satellites, Europa, Ganymede, and Callisto.

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The tour is only one phase of the Europa Orbiter mission. After arriving at Jupiter, a maneuver will be performed to capture the spacecraft about Jupiter in an orbit that encounters Ganymede. Our tours start with variations of this Ganymede encounter. After the tour reduces the final arrival  $V_{\infty}$  at Europa, the endgame begins. The endgame is designed by the Jet Propulsion Laboratory (JPL) to use a combination of Europa flybys, small maneuvers, and 3-body effects to reduce the energy of the orbit further prior to the orbit insertion maneuver (see Johannesen and D'Amario<sup>2</sup>).

# Guidelines and Mission Constraints for Tour Design

We start with a set of initial conditions at Ganymede, which vary depending on when the orbiter is launched from Earth. JPL categorizes these conditions as "beginning," "middle," and "late," launch period. Typical initial conditions from each launch period are given in Table 1. In Table 1, launch period ranges from Nov. 10 to Nov. 25, 2003, and which corresponds to arrival at Jupiter from Feb. 28 to Dec. 5, 2007. Starting from initial conditions such as those in Table 1, we then

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proceed to design the tour subject to various mission guidelines and constraints.

Table 1 Typical initial conditions at Ganymede

Launch	Arrival	$\overline{\mathbf{V}}_{\infty}$	Perijove	Period
Date	Date	(km/s)	$(R_J)$	(days)
Nov. 10,	Feb. 28,	8.18	9.8	200.2
2003 <sup>a</sup>	2007			
Nov. 17,	Jul. 21,	8.47	9.4	199.7
2003 <sup>b</sup>	2007			
Nov. 25,	Dec. 4,	8.14	9.8	191.4
2003°	2007			

a, b, and c correspond to beginning, middle, and late launch period.

There are many constraints that must be met during the tour. Most important is to have low  $V_{\infty}$  at Europa. Originally, JPL set the constraint to a maximum of 3.5 km/s, but lower values are highly desirable.<sup>3</sup> Based on the Hohmann transfer from Ganymede to Europa, the lowest ballistic V<sub>∞</sub> achievable is 1.49 km/s. Periapsis of any orbit in the tour should be greater than 8.8 R<sub>J</sub> (Jovian radii), to mitigate the effects of radiation exposure, which can damage the spacecraft. Flyby altitude at each satellite must be greater than 100 km at each satellite in general, and must be greater than 200 km during the first flyby of any satellite, in order to avoid crashing into the surface due to navigational uncertainties. While in transit between any two satellites, the spacecraft must not approach within 50,000 km of any third body (i.e. a "nontargeted" flyby) in order to avoid perturbing the orbit too much. Another design guideline is to keep the total number of flybys to a minimum, because each flyby may require a slight correctional delta-V. No close flybys are allowed when Jupiter is in solar conjunction because the

Table 2 Maximum arrival V<sub>∞</sub> for a given resonance<sup>3</sup>

Resonance	V <sub>∞</sub> (km/s)
3:1	3.2
5:2	3.6
2:1	3.0
5:3	3.1
4:3	1.8
6:5	1.2

Sun disrupts communication with the spacecraft. Also, the tour should be completed while the spacecraft is within 5 AU of the Earth to maintain a high data rate. The combination of the solar conjunction constraint and the 5 AU constraint limits the time of flight for the tour to a period that varies from roughly 280 to 500 days, depending on whether the tour is from the late, middle or beginning launch period. Each leg of the tour must pass through apoapsis to allow for trajectory correction maneuvers. Finally, each tour must end in a resonant orbit with Europa.

The endgame follows the tour. The endgame consists of a series of Europa flybys combined with a maneuver at apojove. The maneuvers raise perijove and lower  $V_{\infty}$ , while the flybys reduce the period. There is a maximum  $V_{\infty}$  desired for a given final resonance achieved by the tour, as shown in Table 2. For example, for a 4:3 resonance (4 spacecraft revs : 3 Europa revs) the arrival  $V_{\infty}$  at Europa should not exceed 1.8 km/s. On the other hand, a 6:5 resonance requires a  $V_{\infty}$  of less than 1.2 km/s, which is not achievable ballistically. Because it is ballistically possible to achieve less than 1.8 km/s at Europa for the 4:3 resonance, most tours end with a 4:3 resonance.

## **Solution Approach**

The Satellite Tour Design Program (STOUR) is a software tool that was developed by JPL for the Galileo mission tour design.<sup>4</sup> It has been enhanced and extended at Purdue to perform automated design of gravity-assist tours of the Solar System and of the satellite system of Jupiter.<sup>5-8</sup> STOUR uses the patched-conic method to calculate all gravity-assist trajectories meeting specified requirements.

We use STOUR as our principal tool for the design of Europa Orbiter tours. From a starting condition at Ganymede, STOUR finds trajectories for a given path, i.e. a sequence of gravity-assist bodies. The massive number of trajectories produced by STOUR must be sifted through to find viable tour candidates.

Tour 99-02 (the second tour we designed<sup>8,9</sup> in 1999) uses 15 flybys of Europa, Ganymede, and Callisto and is depicted in Fig. 1. Even with the initial conditions specified at Ganymede, there are tens of millions of

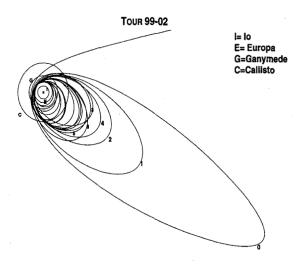


Fig. 1 Baseline tour for Europa orbiter.

possible tours that follow a specified path due to the number of choices of time of flight between encounters. The calculation of these can take weeks for a single path. When we consider that there are  $3^{13}$  (1.6 million) possible paths that begin at Ganymede and reach Europa in 15 encounters, we see that the problem of calculating all possible tours is intractable with current computer technology. Clearly, we need to know what paths have the most promise to yield viable tour candidates before even beginning STOUR computations.

We began tackling this problem by choosing paths by trial and error, tempered with engineering judgement. For instance, we could lower the spacecraft's period with a pump-down flyby and thus decrease the total energy relative to Jupiter in an attempt to reduce the final arrival  $V_{\infty}$  at Europa. A series of pump downs with Ganymede would accomplish this quickly, but would also quickly lower the periapsis into the hazardous radiation environment (i.e. fry the spacecraft). Thus we cannot use Ganymede alone for period reduction. We found that although Europa has less gravity to assist us, it is able to reduce period more than Ganymede for the same decrease in periapsis height. We also noticed that Callisto is handy for raising periapsis, as it can do so with the lowest increase in orbit period. If we combine these satellites in the right order (e.g. Ganymede-Callisto or Ganymede-Europa-Callisto), we could reduce period while maintaining a high enough periapsis at the end of a sequence of satellite flybys. The identification of useful path segments such as these took months of experience to design.

To improve over this trial and error method, we conducted exhaustive searches through all possible five-body path segments for the beginning of the tour. Even limiting the paths to five bodies left us with a computationally intensive and time-consuming process that had to be repeated for each different initial condition at the first Ganymede encounter. Moreover, the results of this endeavor were hard to interpret. A key question is how to characterize what set of five flybys will lead to a good tour. One figure of merit is the  $V_{\infty}$  at the fifth flyby, but it is difficult to draw comparisons between the final  $V_{\infty}$ 's of path segments ending at different satellites (e.g. how do we rate 3 km/s at Ganymede 5 compared to 4 km/s at Europa 5?).

During the initial process we found that tracking both period and periapsis could often identify interesting path segments. Because the satellites we are working with are in almost circular orbits about Jupiter, period and periapsis prescribe both the shape of the spacecraft's orbit about Jupiter and the  $V_{\infty}$  at each satellite.

This observation suggests the "P-r<sub>p</sub>" plot (Fig. 2). This is a plot of period versus perijove for orbits with less than 200 day periods that meet the perijove constraint (≥ 8.8 R<sub>I</sub>). Each point on the plot represents a static orbit about Jupiter. This plot is based on an energy method that does not take into account the time of transfer (phasing). STOUR solves the phasing problem. The plot shows contours of constant V<sub>∞</sub> for each satellite, assuming circular, coplanar orbits. A gravity assist rotates the  $V_{\infty}$ vector of the spacecraft along one of these contours modifying the orbit about Jupiter. Where contours from two satellites intersect, there exists a potential transfer between those satellites. These contours give the values of  $V_{\infty}$  at each satellite for this transfer arc and also provide a method for comparing the  $V_{\infty}$  at different bodies.

If we constrain the flybys to have a minimum altitude of 100 km above the surface of the satellite, we are limited in how far we can travel along a contour in one flyby. This is illustrated on the plot by tick marks (dots). From one tick mark on a contour we may move a maximum of the distance to the next tick mark up or down that contour. (The tick marks also can help us judge how far one flyby can move up or down a contour even when not starting from a tick mark.)

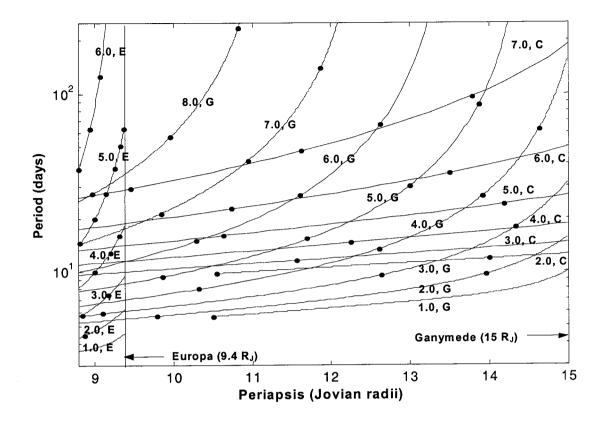


Fig. 2 P-r<sub>p</sub> plot. A single point represents the shape of an orbit around Jupiter. Movement along a  $V_{\infty}$  contour represents the effect of a flyby.  $V_{\infty}$  is in km/s, tick marks (dots) are separated by 100 km flybys. G, C, and E refer to Ganymede, Callisto, and Europa, respectively.

By studying the P-  $r_p$  plot in Fig. 2, we can quickly deduce design concepts that previously took us months to learn. Remembering that our goal with the tours is to decrease the spacecraft's period but still keep the periapsis high, we can see that Europa is most effective in lowering period with a minimal cost in periapsis height by the sharp upward slope of its  $V_{\infty}$  contours. However, due to the distance between the tick marks, Ganymede is much more effective in lowering period with a single flyby. The shallow slope of Callisto's contours show that it is the best choice for raising periapsis, as it costs the least in terms of increased period to do so.

With one of these plots and a pencil, a tour designer can quickly sketch out a promising path for analysis in STOUR. Also, known tours can be plotted and examined for possible improvements.

The P-r<sub>p</sub> plots can be derived from Tisserand's criterion. Tisserand showed in the 19<sup>th</sup> century that comet orbits perturbed by Jupiter's gravity satisfy Jacobi's integral.<sup>10</sup> The resulting equations can be solved and

plotted on a  $P-r_p$  plot. We used Tisserand's criterion to verify our  $P-r_p$  plots.

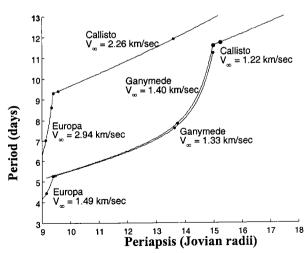


Fig. 3 Hohmann transfers between Ganymede and Europa, Callisto and Europa, and Ganymede and Callisto.

Figure 3 shows the Ganymede-Europa, Callisto-Ganymede, and Callisto-Europa Hohmann transfers. These orbits provide a lower bound of 1.49 km/s for arrival  $V_{\infty}$  at Europa. The plot shows this can only be achieved via multiple Ganymede-Europa arcs at the end of a tour as opposed to directly after a Callisto flyby.

We are currently extending the P- $r_p$  graphical method to search for the shortest time-of-flight path to Europa and for the lowest radiation-dose tour. This involves developing software to automatically traverse the P- $r_p$  plots to find possible paths and calculate a cost for those paths.

#### Results

Altogether, we designed 35 tours in 1999. Tour 99-02 (see Fig. 1) is currently being used as a baseline by JPL. The details of Tour 99-02 are in Table 3.

Tour 99-02 is one of our earlier tour designs, where we relied primarily on trial and error to design and link

Table 3 Tour 99-02 summary

Event #/ Satellite	V <sub>∞</sub> (km/s)	Period (days)	r <sub>p</sub> (R <sub>J</sub> )	Time (days)
1/Ganymede	7.85	64.3	10.3	0
2/Ganymede	7.85	35.7	9.6	64
3/Ganymede	7.86	21.4	8.6ª	100
4/Ganymede	7.86	27.8	9.1	122
5/Europa	5.11	20.4	9.0	151
6/Callisto	6.39	23.1	9.8	169
7/Ganymede	7.10	16.7	9.1	193
8/Europa	4.74	17.7	9.1	211
9/Europa	4.73	16.5	9.1	229
10/Callisto	5.75	22.0	11.4	247
11/Ganymede	5.85	14.3	10.3	268
12/Ganymede	5.85	10.8	9.3	282
13/Europa	3.34	10.6	9.3	303
14/Europa	3.31	8.8	9.1	313
15/Europa	3.29	7.1	8.9	331
16/Europa	3.28			338

<sup>&</sup>lt;sup>a</sup> Constraint violation ( $r_P \ge 8.8 \text{ R}_J$ ) waived by project.

promising path segments. A good example of such a segment is the first 5 flybys of Tour 99-02. We start out with 3 Ganymede resonances, followed by a Europa-Callisto combination. This pattern of multiple Ganymede flybys followed by a Europa-Callisto pairing accounts for the great majority (19) of the tours we designed for the beginning launch period.

Table 4 Tour 99-35 summary

Event #/	$V_{\infty}$	Period	Perijove	Time
Satellite	(km/s)	(days)	$(R_J)$	(days)
1/Ganymede	5.99	50.1	12.5	0
2/Ganymede	5.99	30.5	11.9	50
3/Callisto	6.31	41.9	13.5	84
4/Ganymede	4.93	21.5	12.6	124
5/Ganymede	4.93	13.3	11.4	145
6/Callisto	3.93	18.0	14.9	155
7/Ganymede	2.37	10.7	13.9	194
8/Ganymede	2.37	7.2	11.7	215
9/Ganymede	2.37	5.5	9.1	222
10/Europa	2.45	5.2	9.0	232
11/Ganymede	1.59	5.3	9.4	245
12/Europa	1.64	4.7	9.3	253
13/Europa	1.62			267

For low-radiation tours, we would like the periapsis to remain as high as possible. An orbit with a periapsis above 12 R<sub>I</sub> essentially does not contribute to the radiation hazard.<sup>3</sup> The periapses in Tour 99-02 never exceed 12 R<sub>J</sub>, and are rarely greater than 10 R<sub>J</sub>, because when we designed Tour 99-02 total radiation dose was neither modeled nor constrained. The G3 (Ganymede 3) flyby violated the periapsis constraint by having a lower periapsis than 8.8 R<sub>J</sub> (i.e. 8.6 R<sub>J</sub>). This constraint violation was waived by the project. The flybys of Europa on events 8 and 9 appreciably increase the radiation dosage of Tour 99-02. Because Europa has a semimajor axis of approximately 9.4 R<sub>I</sub>, any flyby of Europa will have a significant radiation dosage. For this reason, our later tours avoid encountering Europa until the end of the tour. However, the early flybys of Europa in Tour 99-02 do serve a purpose. A glance at the P-r<sub>p</sub> plot (Fig. 2) will confirm that Europa can efficiently pump

down the orbital period with only a slight lowering of the periapsis. Tour 99-02 achieves a final  $V_{\infty}$  of 3.28 km/s, which meets the constraint ( $V_{\infty} \leq 3.50$  km/s) imposed by JPL. Later tour designs achieve much lower  $V_{\infty}$ , but at a cost in time of flight.

We used  $P-r_p$  plots to design Tour 99-35. First, a promising path for the tour was selected from the  $P-r_p$  plot and evaluated interactively (in STOUR) to test its effectiveness. We used this run in conjunction with the  $P-r_p$  plot to adjust our selected path as necessary. Finally, the selected path was used as the basis of an automated search in STOUR. A summary of Tour 99-35 is provided in Table 4.

With Tour 99-35, we limited the number of flybys and maintained a high periapsis for low radiation. Consequently, we started Tour 99-35 with the highest periapsis possible. This turned out to be a periapsis of 13.2  $R_J$  for an initial condition from the late launch period. The use of the P-r<sub>p</sub> plot paid off nicely, as Tour 99-35 has the lowest time of flight of any tour we designed. Tour 99-35 also has a final arrival  $V_\infty$  of 1.62 km/s, which is fairly close to the Hohmann limit of 1.49 km/s. The radiation dosage during Tour 99-35 is minimal through event 10, and if we had ended the tour on event 10, Tour 99-35 would have an exceptionally low-radiation

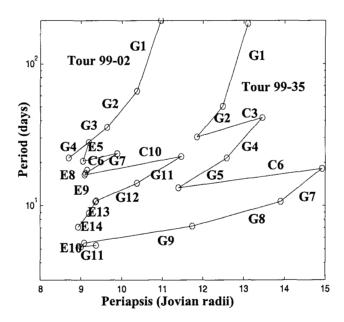


Fig. 4 P-r<sub>p</sub> comparison of Tours 99-02 and 99-35.

dosage. However, we chose to append an additional Ganymede-Europa sequence to lower the final arrival  $V_{\infty}$  from 2.45 km/s to 1.62 km/s (a considerable

improvement). Consequently, we take a hit in radiation dose on events 11 and 12.

The respective paths of Tours 99-02 and 99-35 appear in the P- rp in Fig. 4. A comparison of these tours demonstrates the efficacy of the P-r<sub>p</sub> plot. From the point of view of path selection, we can see a clear inefficiency in Tour 99-02 for events G3 and G4. G3 pumps all the way down to a periapsis of 8.6 (which is a slight violation of the r<sub>p</sub> constraint), and then G4 pumps up to a transfer to Europa (E5). Instead of this roundabout method of reaching E5, in retrospect we could have simply used the G3 transfer to reach Europa, thus saving a flyby and reducing the radiation dose. A similar inefficiency for Tour 99-02 occurs with the E8 and E9 flybys. On the other hand, Tour 99-35 proceeds smoothly from initial condition to final arrival. There is very little "wasted movement" or meandering about the P-r<sub>p</sub> plot. Furthermore, in general each flyby in Tour 99-35 moves farther along a  $V_{\infty}$  curve than the flybys of Tour 99-02, implying more efficient use of each flyby. Thus, the P-rp plot not only aids in designing a tour, but it also provides a means of critiquing a final tour design.

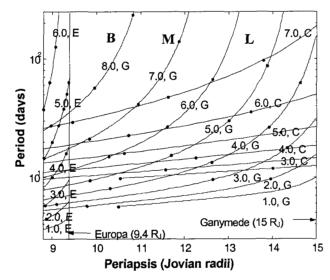


Fig. 5  $P-r_P$  plot of sample initial conditions. B, M, and L correspond to the beginning, middle, and late launch periods, respectively.

Tour 99-35 also benefits from having a better initial condition. When we designed Tour 99-35, our goal was a lower radiation dosage, so we selected the highest initial  $r_p$  available. Sample initial conditions for tour design are

plotted in the P-r<sub>p</sub> plot in Fig. 5. The beginning launch period initial conditions are marked with a "B"; the middle launch period initial conditions with an "M"; and the late launch period initial conditions with an "L". By comparing the positions on the P-r<sub>p</sub> plot of the beginning, middle, and late launch periods, we can see that the initial GGEC sequence for the B tours makes sense. Given the low periapsis, we need to pump down quickly to get to one of the shallow Callisto V<sub>∞</sub> curves, which are efficient at increasing periapsis while not increasing period too much. We can also see that because the M and L tours start with higher rp values, we can reduce period somewhat more at the beginning of the tour without lowering the periapsis too much (and thus our radiation dosage is much lower). (Of course, there is a delta-V cost associated with starting the tour at a higher periapsis). Also, the time of flight for the L and M tours will be generally lower, because they start with a lower  $V_{\infty}$  value. Clearly, the initial conditions greatly affect our tour design strategy.

Given the combination of  $V_{\infty}$  and low-radiation constraints, we almost always want our last Callisto-Ganymede transfer orbit to have a periapsis as close to Ganymede's semimajor axis  $(14.97~R_J)$  as possible (because we are trying to achieve a Hohmann transfer between Callisto and Ganymede). In practice, due to phasing, the ideal transfer between Ganymede and Callisto proves elusive, as does the final Ganymede-Europa transfer. In fact, the final sequence of flybys is much more of a limiting factor than any other portion of the tour (i.e., in the middle of the tour, many transfer orbits for a given flyby are available; but at the end, only a few).

Table 5 Details of best tours

Tour (Launch Period)	V <sub>∞</sub> (km/s) <sup>a</sup>	Duration (days)	Rad. Dose <sup>b</sup>
Tour 99-18	1.74	436	9.2
(Beginning)			
Tour 99-26	1.92	386	8.4
(Middle)			
Tour 99-35	1.62	253	8.2
(Late)			

a At Europa.

Table 5 lists the best tours for arrival  $V_{\infty}$ , time of flight, radiation dose, and number of flybys for each launch period. This includes Tour 99-35 of course, which is the best for the late launch period. Tour 99-18 and Tour 99-26 provide excellent missions for the beginning and middle launch periods respectively. Because of the earlier arrival dates, these two tours have significantly longer duration. All these tours satisfy the flyby altitude, non-targeted, and solar conjunction constraints. They provide low-radiation dose (down 70% from the dose of Tour 99-02) and low arrival  $V_{\infty}$ .

The design concepts developed in this paper will dramatically improve future tour designs for the tour launch window.

#### **Conclusions**

The Europa Orbiter mission presents new challenges in mission design because of the enormous number of possible tours. The automated gravity-assist design technique developed in earlier work proved ineffective (by itself) against this computationally gigantic task. Experience through trial and error, and the identification of some rules of thumb provided inroads into the problem and resulted in a baseline, flyable tour. A breakthrough came with the discovery of a graphical method based on Tisserand's criterion. The graphical method led to great improvements in the  $V_{\infty}$  of arrival at Europa (which was reduced from 3.3 to less than 2 km/s) and in the total radiation dosage (which was reduced by 70%). These results exceeded the expectations of mission designers at JPL. Now we have a theory that will guide all future tour design and that will have clear applications in future gravity-assist missions in the Solar System.

This theory has streamlined tour design for the Europa Orbiter mission so that new tours with particular characteristics (such as flight time, low-radiation dose, and fewest flybys) can be quickly designed. This is particularly important because the launch date for the Europa Orbiter mission has been slipped to 2006.

Without a theory, such a slip could be devastating to mission designers. Yet, we welcome the opportunity to demonstrate the power of this tool to design new Europa Orbiter tours and to design new missions for the exploration for the Solar System.

b Normalized to one dose at 9.4 R<sub>J</sub>.

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